Suggested solutions to the Contract Theory exam on Jan. 13, 2010
VERSION: 3 February 2010

Question 1

a) **Explain what is meant by "bunching" (also called "pooling") in an adverse selection model. Argue with the help of a graphical analysis (and under the simplifying assumption that the agent's indifference curves are linear) that bunching is never optimal in a standard two-type adverse selection model. Can bunching occur in a three-type adverse selection model (no proof or argument is required)?**

- The meaning of "bunching" (or "pooling") is that two or more types are served but with the same contract – that is, they are asked to do the same action and the get the same payment.
- Why is bunching not optimal in a two-type model? If the principal bunches, then: (i) the IC-constraints are trivially satisfied; and (ii) IR-bad implies IR-good. Thus the single contract should maximize the principal's profit subject only to IR-bad. Bundle B in the figure (L4-I, fig 1 — attached at the end of this document) indicates the optimum. At B, the slope of the principal's indiff curve equals $\overline{\theta}$. Claim: The principal would benefit from separating the two types by adding a bundle A aimed at the good type; in particular, A can be located just right of B along the good types indiff curve through B (see L4-I, fig 2). To understand why this claim is true, note that offering A and B is *feasible*: the bundles satisfy the ICs and the IRs. Moreover, offering A and B *yields higher profit* than offering only B. For:
  - The profits coming from the bad type are the same as before.
  - And the profits from the good type are higher, as the move from B to A must lead to a better indifference curve [for the path along the arrows has slope $\underline{\theta}$, and the slope of the indiff curve through B at B has slope $\overline{\theta}$].
- **Conclusion**: (i) If serving both types with a single bundle, the best bundle is B. (ii) Only offering B cannot be optimal. (iii) Therefore bunching cannot be optimal.
- **Three types**: Yes, with three types bunching can occur.

b) **Explain in words what the revelation principle is and why it is a useful result.**

- When we are solving a contract theory model we (implicitly) assume that the only instrument available to the principal is a so-called direct revelation mechanism. That is, the principal offers a menu of contracts from which the agent can choose from, where the number of contracts in the menu being equal to the number of agent types. Then the agent is simply asked to announce his type, after which he is assigned a contract according to his announcement.
- When solving the contract theory problem we want to be sure that we identify the contract that indeed is optimal for the principal. In particular, we would like to know whether our assumption that a direct revelation mechanism is used is restrictive — could the principal possibly be better off if he instead used some other, more complicated way of making the agent reveal his type? The revelation principle says that this is not the case: if an outcome is possible to implement at all, a direct revelation mechanism can do the job.
- The revelation principle is a useful result because of the reason mentioned above: Thanks to the revelation principle we know that the optimal contract that we derive is optimal in a very broad class of possible contracts.
- The above is enough for "full score" on this sub-question. However, in the lecture slides the following two points were also mentioned:
  - The revenue equivalence theorem in auction theory makes use of the revelation principle.
  - The Myerson-Satterthwaite theorem (about the impossibility of an efficient outcome in bilateral bargaining with private information about the valuations) makes use of the revelation principle.

c) **Explain briefly the design of the experiment that is reported in Anderhub, Gächter and Königstein (Experimental Economics, 2002). In the paper, the authors make a list of nine "observations" that summarize their experimental results. Give a brief account of five of these observations.**

- The design:
  - The subjects were divided into a group of principals and a group of agents. They were then paired (principal-agent), and each pair played a contract offer game six times. Thereafter new pairs were formed and these played the game (with the same opponent) six more times.
  - The contract offer game works as follows. First the principal chooses a contract, consisting of a fixed wage f (multiples of one, between -700 and 700) and a return share s (multiples of 0.01, between 0 and 1). Then the agent receives this contract and is asked to accept or reject. Finally, if accepting, the agent chooses

an effort level e (multiples of one, between 0 and 20). Choosing a positive effort level leads to a cost for the agent, according an increasing and convex function shown in table form. However, a larger effort also leads to a larger output — this relationship is also shown in table form.

  - In addition, in order to measure the principal's intentions when designing a particular contract, the principal was asked to *suggest* effort levels to the agents. However, the suggestions were not at all binding, so they shouldn't have any strategic significance (or so Anderhub et al seem to argue).
  - The agent received a payoff equal to his fixed fee plus his share (s) of the output minus the effort cost.
  - The principal received a payoff equal to minus the fee plus his share $(1 - s)$ of the output.

- The observations:
  1. In most cases the offered return share s is in the range predicted by the subgame perfect equilibria.
  2. Principals aim at inducing efficient (=the highest possible) effort. [More than 82% suggested the highest effort level.]
  3. Most contracts exhibit a negative fixed wage.
  4. "Selling the firm" (i.e., $s = 1$) occurs in about 30% of the cases. Roughly 70% of all contracts are of mixed type (i.e., $0 < s < 1$). Of those, at least a quarter contains positive fixed wages.
  5. Almost all offered contracts satisfy the individual rationality constraint. [Only 13 out of 564 did not.]
  6. Surplus sharing suggested by principals is less asymmetric—i.e., more fair—than predicted by the theoretical solution.
  7. The agents' acceptance decisions support the *Fairness-Hypothesis*, which says that "The influence of the agent's surplus share on her acceptance rate is positive". [Anderhub et al (2002) conclude this after having run logit regressions (the binary acceptance decision against the size of the agent's surplus share and a constant).]
  8. Agents' effort choices support the "Rational-Effort Hypothesis" — meaning that there is a substantial proportion of best reply effort levels. [There is an even larger number of efficient effort choices (i.e., $e = 20$ also when 20 is not a best reply).]
  9. The deviations from the conditionally rational effort levels are compatible with the idea of positive and negative reciprocity.

d) **In the part of the course that was based on the paper by Vickers (Oxford Economic Papers, 1995) we studied a model with no explicit incentives but where the agent was disciplined by implicit incentives. Explain in words the logic of that model (the single-agent case suffices).**

- In this model the agent is not disciplined by a written contract. Instead the model captures the idea that an agent may care about how he/she is perceived by future potential employers, and therefore want to work hard and thereby create a good output, hoping that the employers who observe that output will infer that the agent is able and worth a high salary.
- In the formal model it is assumed that there are at least two time periods. Moreover, the agent is assumed to have an ability (or productivity) that is unknown (even for himself, actually), and this ability stays the same over time. Therefore the agent has an incentive to work hard in early periods in order to produce a high output, which is more likely to occur if the agent truly is an able agent. The potential employers observe the agent's output and uses this information to update their beliefs about the agent's ability. To solve the model one has to identify a Nash equilibrium in the game between the agent (who's choosing his effort) and the employee (who's choosing what to believe). At the equilibrium the employer is not fooled, but correctly believes that the agent chooses the equilibrium effort level. Nevertheless, the agent's effort higher than in a static model without any kind of incentives (this effort level is zero).

Question 2 (adverse selection)

Consider the following model of a market for pencils that can be produced in different qualities. There are a continuum of consumers (the "agent" of the adverse selection model), each of whom purchasing either one pencil or no pencil. A fraction $\nu \in (0, 1)$ of the consumers have a high valuation for pencil quality and the remaining fraction $(1 - \nu)$ have a low valuation for pencil quality (and the total number of consumers is normalized to one). The high-valuation consumers' payoff if consuming one pencil of quality $\overline{q}$ at the price $\overline{t}$ is given by

$$\overline{\theta}\overline{q} - \overline{t},$$

where $\overline{\theta} > 0$ is a parameter. The low-valuation consumers' payoff if consuming one pencil of quality $\underline{q}$ at the price $\underline{t}$ is given by

$$\underline{\theta}\underline{q} - \underline{t},$$

where $\underline{\theta}$ is a parameter satisfying $\overline{\theta} > \underline{\theta} > 0$. If the consumers (both the high- and low-valuation ones) choose not to consume any pencil at all, their payoff is zero. There is a firm (the "principal" of the adverse selection model) that has a monopoly in the pencil market. If selling one pencil of quality $\underline{q}$ to each of the low-valuation consumers and one pencil of quality $\overline{q}$ to each of the high-valuation consumers, the firm incurs the production costs

$$\frac{1-\nu}{2}\underline{q}^2 + \frac{\nu}{2}\overline{q}^2.$$

The firm's total profits are therefore given by

$$(1-\nu)\,\underline{t} + \nu\overline{t} - \frac{1-\nu}{2}\underline{q}^2 - \frac{\nu}{2}\overline{q}^2.$$

Each consumer knows his or her own θ perfectly. However, the monopoly firm does not know the θ of an individual consumer, but only that a fraction ν of the consumers have a high valuation and that the rest have a low valuation. The objective of the firm is to maximize its total profits.

a) **Suppose the parameters are such that the firm optimally interacts with both kinds of consumers. Formulate the optimization problem that the firm faces when designing the menu of prices: state the objective function and the constraints, and explain what the choice variables are. Explain the meaning of the constraints in words.**

- The firm's objective function is given by its total profits:

$$(1-\nu)\,\underline{t}+\nu\bar{t}-\frac{1-\nu}{2}\underline{q}^2-\frac{\nu}{2}\overline{q}^2.$$

The firm wants to maximize that expression with respect to the choice variables $\underline{t}$, $\bar{t}$, $\underline{q}$, and $\overline{q}$, subject to the following four constraints:

  – The low-valuation customers must prefer their bundle to no bundle at all (individual rationality for the L-type):

$$\underline{\theta}\underline{q}-\underline{t}\geq 0. \tag{IR-L}$$

  – The high-valuation customers must prefer their bundle to no bundle at all (individual rationality for the H-type):

$$\overline{\theta}\overline{q}-\bar{t}\geq 0. \tag{IR-H}$$

  – The low-valuation customers must prefer their bundle to the high-valuation customers' bundle (incentive compatibility for the L-type):

$$\underline{\theta}\underline{q}-\underline{t}\geq \underline{\theta}\overline{q}-\bar{t}. \tag{IC-L}$$

  – The high-valuation customers must prefer their bundle to the low-valuation customers' bundle (incentive compatibility for the H-type):

$$\overline{\theta}\overline{q}-\bar{t}\geq \overline{\theta}\underline{q}-\underline{t}. \tag{IC-H}$$

b) **Prove formally that any pair of qualities $\left(\underline{q},\overline{q}\right)$ that satisfy the constraints under a) also satisfy $\underline{q}\leq\overline{q}$. Illustrate the argument of the proof in a diagram with q and t on the axes.**

- To prove this we need only two of the four constraints, namely (IC-L) and (IC-H). Adding these constraints yields

$$\left(\underline{\theta}\underline{q}-\underline{t}\right)+\left(\overline{\theta}\overline{q}-\bar{t}\right)\geq\left(\underline{\theta}\overline{q}-\bar{t}\right)+\left(\overline{\theta}\underline{q}-\underline{t}\right)$$

or, since the transfers cancel out,

$$\underline{\theta}\underline{q}+\overline{\theta}\overline{q}\geq\underline{\theta}\overline{q}+\overline{\theta}\underline{q}.$$

Rewriting this, we have

$$\left(\overline{\theta}-\underline{\theta}\right)\left(\overline{q}-\underline{q}\right)\geq 0.$$

But since $\overline{\theta}-\underline{\theta}>0$ by assumption, the last inequality simplifies to $\overline{q}-\underline{q}\geq 0$, which we were supposed to prove. That is, the two incentive compatibility constraints imply monotonicity ($\underline{q}\leq\overline{q}$). More generally, we know from the course that in adverse selection models monotonicity is implied by the IC constraints and the Spence-Mirrlees (or single-crossing) condition. Here, however, the Spence-Mirrlees condition is implicit in our chosen functional forms.

- For the graphical illustration, see attached figure (L3-I, fig 2). This should be explained (for some partial explanation of the argument, see the text in the figure).

c) **Let the first-best levels of $\underline{q}$ and $\overline{q}$ be defined as the ones that maximize the total surplus,**

$$(1-\nu)\,\underline{\theta}\underline{q}+\nu\overline{\theta}\overline{q}-\frac{1-\nu}{2}\underline{q}^2-\frac{\nu}{2}\overline{q}^2.$$

Calculate these first-best levels. Explain the economic intuition behind your result.

- It is stated in the question that the first best levels are defined as the ones that maximize the above expression for the total surplus. To calculate these we can take the first-order conditions with respect to $\underline{q}$ and $\overline{q}$. Doing that yields

$$\frac{\partial}{\partial \underline{q}}\left[(1-\nu)\,\underline{\theta}\underline{q}+\nu\overline{\theta}\overline{q}-\frac{1-\nu}{2}\underline{q}^2-\frac{\nu}{2}\overline{q}^2\right]=(1-\nu)\,\underline{\theta}-(1-\nu)\,\underline{q}=0\Rightarrow \underline{q}^{FB}=\underline{\theta}$$

and

$$\frac{\partial}{\partial \overline{q}}\left[(1-\nu)\,\underline{\theta}\underline{q}+\nu\overline{\theta}\overline{q}-\frac{1-\nu}{2}\underline{q}^2-\frac{\nu}{2}\overline{q}^2\right]=\nu\overline{\theta}-\nu\overline{q}=0\Rightarrow \overline{q}^{FB}=\overline{\theta}.$$

(The second-order condition is clearly satisfied as the objective is quadratic in the choice variables and the coefficients for the quadratic terms are negative.)

- The intuition is that these quantities are the ones that, given a known value of θ, ensure that the agent's marginal benefit of consuming the good (MB) is equal to the principal's marginal cost of producing the good (MC). If we had MB$\neq$MC, total surplus wouldn't be maximized.

d) **Now return to the second-best problem you have formulated under a). Solve this problem. Explain how the optimal second-best qualities differ from the optimal first-best qualities. Also explain the economic intuition behind any differences. Which type, if any, gets any rents at the second-best optimum? Why?**

- We can solve the problem by making use of the following five-step recipe:

  1 Show that IR-L and IC-H imply IR-H, so we can ignore IR-H.

  2 *Guess* that IC-L doesn't bind.

  3 Inspect the problem and note that the two remaining constraints must bind. Therefore we can plug them into the objective function.

4 Solve the resulting unconstrained problem.

5 Verify that the solution satisfies IC-L (i.e., that the guess at (2) was correct).

- The claim that IR-L and IC-H imply IR-H can be proven as follows:

$$\overline{\theta}\overline{q} - \overline{t} \geq \overline{\theta}\underline{q} - \underline{t} > \underline{\theta}\underline{q} - \underline{t} \geq 0. \tag{1}$$

The first inequality is the same as IC-H. The second inequality follows from the assumption that $\overline{\theta} > \underline{\theta}$ (and the fact that $\underline{q} > 0$). The third inequality is the same as IR-L. The above sequence of inequalities means that $\overline{\theta}\overline{q} - \overline{t} \geq 0$, which is the same as IR-H, so we have proven the claim.

- If we also guess that IC-L doesn't bind, the remaining constraints are IR-L and IC-H:

$$\underline{\theta}\underline{q} - \underline{t} \geq 0, \tag{IR-L}$$

$$\overline{\theta}\overline{q} - \overline{t} \geq \overline{\theta}\underline{q} - \underline{t}. \tag{IC-H}$$

- The objective is decreasing in $\underline{t}$ and $\overline{t}$. Therefore if one or both of the constraints did not bind, the principal would be able to increase his payoff. That is, the two constraints must both bind at the optimum.
- Setting the constraints to equality and solving for $\underline{t}$ and $\overline{t}$ yield

$$\underline{t} = \underline{\theta}\underline{q}, \tag{2}$$

$$\begin{aligned} \overline{t} &= \overline{\theta}\overline{q} - \overline{\theta}\underline{q} + \underline{t} \\ &= \overline{\theta}\overline{q} - \left(\overline{\theta} - \underline{\theta}\right)\underline{q} \end{aligned} \tag{3}$$

- Plugging into the objective:

$$\begin{aligned} \pi &= (1-\nu)\,\underline{t} + \nu\overline{t} - \frac{1-\nu}{2}\underline{q}^2 - \frac{\nu}{2}\overline{q}^2 \\ &= (1-\nu)\,\underline{\theta}\underline{q} + \nu\left[\overline{\theta}\overline{q} - \left(\overline{\theta} - \underline{\theta}\right)\underline{q}\right] - \frac{1-\nu}{2}\underline{q}^2 - \frac{\nu}{2}\overline{q}^2 \end{aligned}$$

- The first-order conditions:

$$\frac{\partial\pi}{\partial\underline{q}} = (1-\nu)\,\underline{\theta} - \nu\left(\overline{\theta} - \underline{\theta}\right) - (1-\nu)\,\underline{q} = 0 \Rightarrow \underline{q}^{SB} = \underline{\theta} - \frac{\nu\left(\overline{\theta} - \underline{\theta}\right)}{1-\nu} \tag{4}$$

$$\frac{\partial\pi}{\partial\overline{q}} = \nu\overline{\theta} - \nu\overline{q} = 0 \Rightarrow \overline{q}^{SB} = \overline{\theta}. \tag{5}$$

- We also need to show that IC-L is satisfied at the (possible) solution we have found:

$$\underline{\theta}\underline{q}^{SB} - \underline{t}^{SB} \geq \underline{\theta}\overline{q}^{SB} - \overline{t}^{SB} \tag{IC-L}$$

or (using (2) and (3))

$$\underline{\theta}\underline{q}^{SB} - \underline{\theta}\underline{q}^{SB} \geq \underline{\theta}\overline{q}^{SB} - \left[\overline{\theta}\overline{q}^{SB} - \left(\overline{\theta} - \underline{\theta}\right) \underline{q}^{SB}\right] \qquad \text{(IC-L)}$$

or

$$\left(\overline{\theta} - \underline{\theta}\right) \left(\overline{q}^{SB} - \underline{q}^{SB}\right) \geq 0 \qquad \text{(IC-L)}$$

or (because $\overline{\theta} - \underline{\theta} > 0$)

$$\overline{q}^{SB} \geq \underline{q}^{SB} \qquad \text{(IC-L)}$$

or (using (4) and (5))

$$\overline{\theta} \geq \underline{\theta} - \frac{\nu\left(\overline{\theta} - \underline{\theta}\right)}{1 - \nu}, \qquad \text{(IC-L)}$$

which clearly is satisfied (as $\overline{\theta} > \underline{\theta}$). We conclude that IC-L is satisfied at the (possible) solution, this is indeed the solution.

- We thus have $\overline{q}^{SB} = \overline{q}^{FB}$ (efficiency/no distortion at the top) and $\underline{q}^{SB} < \underline{q}^{FB}$ (inefficiency/distortion at the bottom).
- We also conclude that the L-type does not get any rents (i.e., any utility on top of what that agent gets for his outside option), as IR-L binds. However, IR-H is satisfied with a strict inequality at the optimum — this follows already from (1). So we have rent extraction at the bottom but not at the top.
- **Intuition:** Key to the results is that the high type is the one who gets, for any given q, both: (i) the highest *marginal* utility [the "single-crossing condition"] and (ii) the highest *total* utility.
- Because of (ii), the firm primarily wants to extract the high type's surplus (as it's larger).
  - However, if the high type gets too little, he can choose the low type's bundle instead.
- To prevent this, the monopolist makes the low type's bundle less attractive by offering those consumers less.
- This works because of (i): The high type suffers more from a reduction in q than the low type.
  - See the attached figure [L2-II, fig 4].
- Suppose q stands for quality and the firm is a railway company.
  - Then the difference in service level between first- and second-class is larger under second best than under first best:

q-distance under FB

$$\underline{q}^{SB} < \underline{q}^{FB} < \overline{q}^{SB} = \overline{q}^{FB}$$

q-distance under SB

- The first-class service level is the same under first and second best, whereas the second-class service level is distorted downwards.
- The intuitive reason: The intended first-class passengers mustn't want to buy second-class tickets instead, so let's make second class sufficiently uncomfortable!

Question 3 (moral hazard)

Prometheus Sørensen (the principal, P for short) owns a factory producing pencils and wants to hire Absalon Nielsen (the agent, A for short) to work there. If hired, A's task will be to operate a pencil machine and to make sure it runs smoothly. To do this well, A must "make an effort", which involves a (personal) cost to A. This is modelled as A's choosing an effort level $e \in \{0, 1\}$, where $e = 1$ means "making an effort" and $e = 0$ means "not making an effort". The associated cost equals

$$\psi(e) = \begin{cases} \psi & \text{if } e = 1 \\ 0 & \text{if } e = 0, \end{cases}$$

with $\psi > 0$. The number of pencils that come out of the machine, q, is either large ($q = \overline{q}$) or small ($q = \underline{q}$), with $\overline{q} > \underline{q} > 0$. The probability that the number is large depends on whether A has made an effort or not:

$$\Pr(q = \overline{q} \mid e) = \begin{cases} \pi_1 & \text{if } e = 1 \\ \pi_0 & \text{if } e = 0, \end{cases}$$

with $0 < \pi_0 < \pi_1 < 1$. P (and the court) can observe which quantity that is realized ($\overline{q}$ or $\underline{q}$) but not the effort level chosen by A. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A. A contract can specify two numbers, $\overline{t}$ and $\underline{t}$, where $\overline{t}$ is the payment to A if $q = \overline{q}$, and $\underline{t}$ is the payment to A if $q = \underline{q}$. P is risk neutral and his payoff, given a quantity q and a payment t, equals

$$V = q - t.$$

A is also risk neutral and his payoff, given a payment t and an effort level e, equals

$$U = t - \psi(e).$$

A is protected by limited liability, meaning that $\overline{t} \geq 0$ and $\underline{t} \geq 0$. A's outside option would yield the payoff $\widehat{U} \geq 0$.

a) **Assume that $\widehat{U} = 0$. Calculate (analytically, not using a figure) P's cost of implementing the high effort level when (i) P can observe A's effort (i.e., the first best) and (ii) when P cannot observe A's effort (i.e., the second best). Compare these costs and explain in what sense effort is underprovided in the model due to asymmetric information. How would the conclusion change if $\pi_0 = 0$? Explain the economic significance of the assumption that $\pi_0 > 0$.**

- To implement a high effort when the effort is observable will cost

$$C^{FB} = \psi.$$

This is the cost that the agent himself incurs when making a high effort. The principal can write into the contract that the agent must exert a high effort (as the effort is observable), but as compensation the principal must pay at least ψ for the agent to accept the contract (as the outside option gives utility zero). However, the principal does not need to pay more than that (if being paid ψ the agent is indifferent between accepting and rejecting, and so there is an equilibrium in which he does accept — the convention in the contract theory literature is to focus on that equilibrium).

- To implement a high effort when effort is not observable, the principal should solve the following problem:

$$\max_{\bar{t},\underline{t}} \left\{\pi_1 \left(\bar{q} - \bar{t}\right) + (1 - \pi_1) \left(\underline{q} - \underline{t}\right)\right\} \text{ subject to}$$

$$\pi_1 \bar{t} + (1 - \pi_1) \underline{t} - \psi \geq \pi_0 \bar{t} + (1 - \pi_0) \underline{t} \Leftrightarrow (\pi_1 - \pi_0) \left(\bar{t} - \underline{t}\right) - \psi \geq 0 \tag{IC}$$

$$\pi_1 \bar{t} + (1 - \pi_1) \underline{t} - \psi \geq 0, \tag{IR-H}$$

$$\underline{t} \geq 0 \qquad \text{and} \qquad \bar{t} \geq 0. \tag{LL-L and LL-H}$$

- Since $\psi > 0$ and $\pi_1 - \pi_0 > 0$, IC implies that $\bar{t} > \underline{t}$, which in turn means that LL-H must be lax.
- Moreover, IC and the two LL-constraints imply IR-H, so we can ignore IR-H.
- The Lagrangian:

$$\mathcal{L} = \pi_1 \left(\overline{S} - \bar{t}\right) + (1 - \pi_1) \left(\underline{S} - \underline{t}\right) + \lambda \left[(\pi_1 - \pi_0) \left(\bar{t} - \underline{t}\right) - \psi\right] + \xi \underline{t}$$

- FOC w.r.t. $\bar{t}$:

$$\frac{\partial \mathcal{L}}{\partial \bar{t}} = -\pi_1 + \lambda (\pi_1 - \pi_0) = 0,$$

  which immediately shows that IC binds as $\lambda > 0$.
- FOC w.r.t. $\underline{t}$:

$$\frac{\partial \mathcal{L}}{\partial \underline{t}} = -(1 - \pi_1) - \lambda (\pi_1 - \pi_0) + \xi = 0$$

- Adding up the two FOCs yields

$$\xi = 1, \tag{6}$$

  which means that LL-L must be binding.
- We thus know that IC and LL-L bind. The latter means that

$$\underline{t}^{SB} = 0,$$

  and plugging that expression for $\underline{t}^{SB}$ into the binding IC yields

$$\bar{t}^{SB} = \frac{\psi}{\pi_1 - \pi_0}.$$

- The cost of implementing the high effort level when effort is not observable is thus

$$C^{SB} = \pi_1 \bar{t}^{SB} + (1 - \pi_1)\, \underline{t}^{SB} = \frac{\pi_1 \psi}{\pi_1 - \pi_0}.$$

- Simple algebra shows that

$$C^{SB} > C^{FB} \Leftrightarrow \pi_0 > 0,$$

  which is satisfied under our assumptions. This means that the cost of implementing the high effort is higher when effort is unobservable compared to when it is observable. Therefore, there will be some parameter values (or, some levels of the benefit of implementing the high effort) for which the high effort is implemented under first best but not under second best — that is the sense in which effort will be underprovided due to asymmetric information.

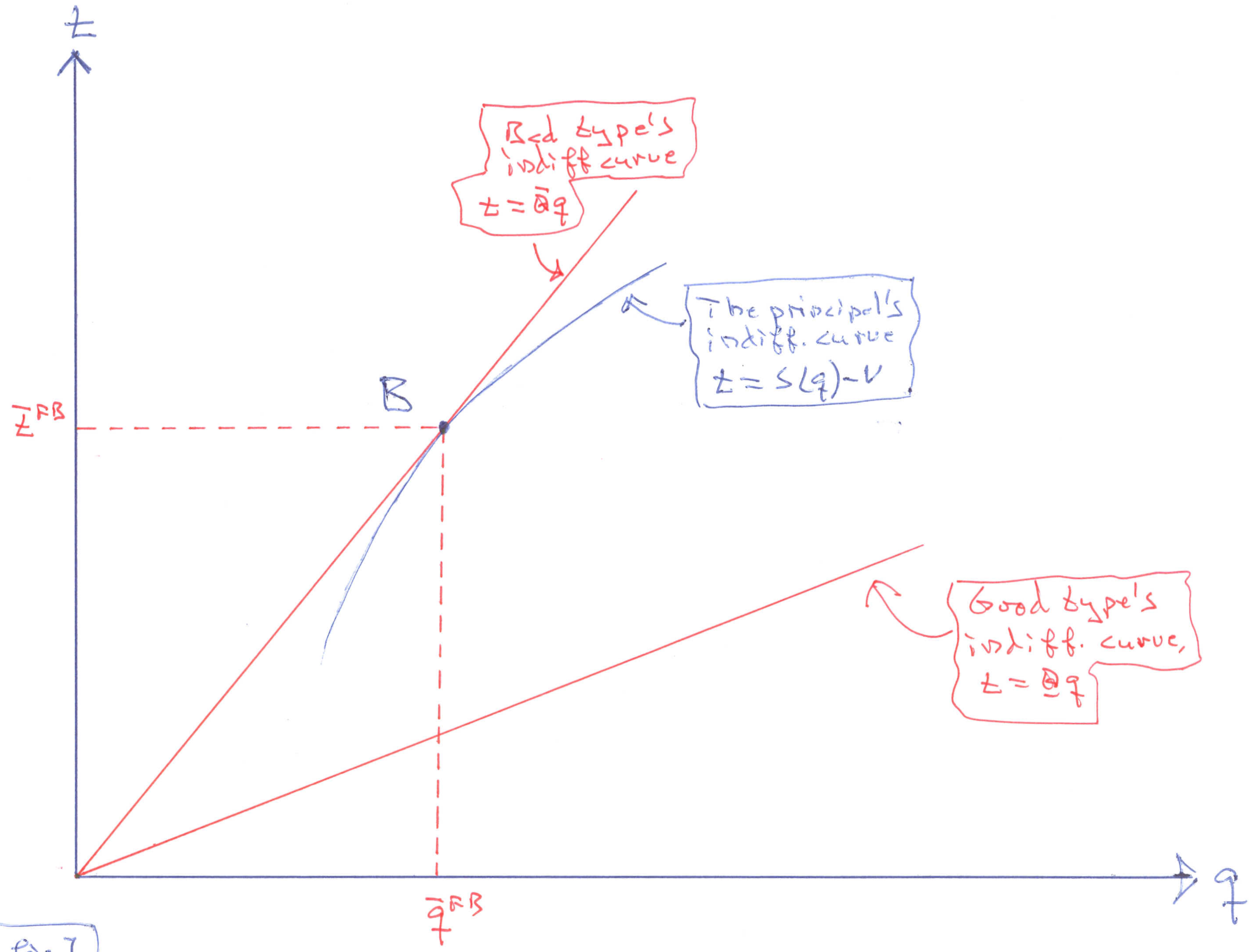
- If $\pi_0 = 0$, however, we would have $C^{SB} = C^{FB}$ and no underprovision result.
  - To understand this, note that the way the principal, under first best, implements a high effort at cost ψ is to pay ψ if and only if a high effort is observed. Under second best the effort is not observable, so that contract is not feasible. However, the principal can pay ψ if and only if the good outcome is observed. That means that, given $\pi_0 = 0$, the agent only gets rewarded if making a high effort, so the principal can effectively do what he's doing under first best.
  - One can also think in terms of what the principal can infer about the agent's effort choice after having observed a good outcome. As $\pi_0 = 0$ means that the good outcome occurs with probability zero if exerting a low effort, a good outcome is an indication that the agent *must* have exerted a high effort. So, when it matters (namely, when the good outcome — the one the principal wants to implement — is observed) the principal effectively can observe the effort choice as long as $\pi_0 = 0$.

b) **Relax the assumption that $\widehat{U} = 0$ and allow for any $\widehat{U} \geq 0$. Only consider the case where P wants to induce A to make an effort. Illustrate the second-best solution in a diagram with $\bar{t}$ on the vertical axis and $\underline{t}$ on the horizontal axis. Show in the figure and explain, in qualitative terms, how the nature of the second best solution changes as the outside option utility $\widehat{U}$ becomes larger.**

- For $\overline{U}$ positive and large enough (in particular, for $\overline{U} \geq \frac{\pi_0}{\pi_1 - \pi_0}\psi$) the IR-H constraint becomes binding and the optimal solution is any combination of $\bar{t}$ and $\underline{t}$ such that IR-H binds and both LL-L and IC are satisfied. In terms of a figure (L7, fig 2 — attached at the end of this document), this can be illustrated by moving the graph of IR-H north-east in a parallel fashion until it goes through the original feasible set. The optimal transfer levels are then the ones on the IR-H line and still within the original feasible set.

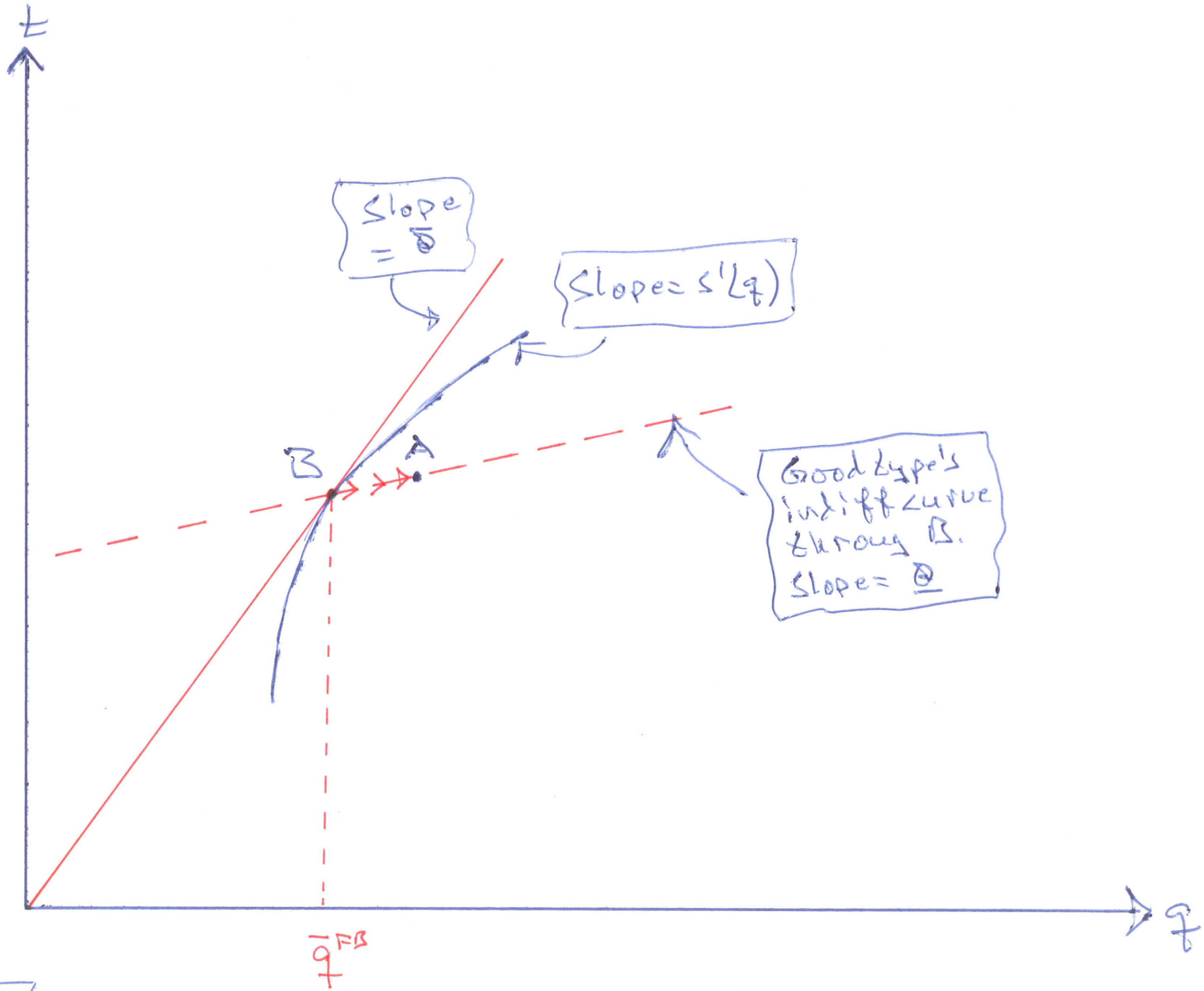
c) **Suppose that the agent is not protected by limited liability. Explain in words how and why this affects the nature of the second-best solution.**

- In this case the second-best solution will not involve an inefficiency (e.g., it coincides with the first-best solution).
- The economic meaning of the fact that the agent is risk neutral is that he cares only about whether his payment t is large enough *on average*. Hence, the principal can, without violating the individual rationality constraint, incentivize the agent by giving him a negative payment (in practice a penalty) in case of a low output. More generally, the principal can achieve the first-best outcome by making the agent the residual claimant:
  - The agent effectively buys the right to receive any returns ($\overline{q}$ or $\underline{q}$): "the firm is sold to the agent".
  - Thereby, the effort level is chosen by the same individual who bears the consequences of the choice.
  - In this situation the agent makes the same effort choice as the principal would have made.

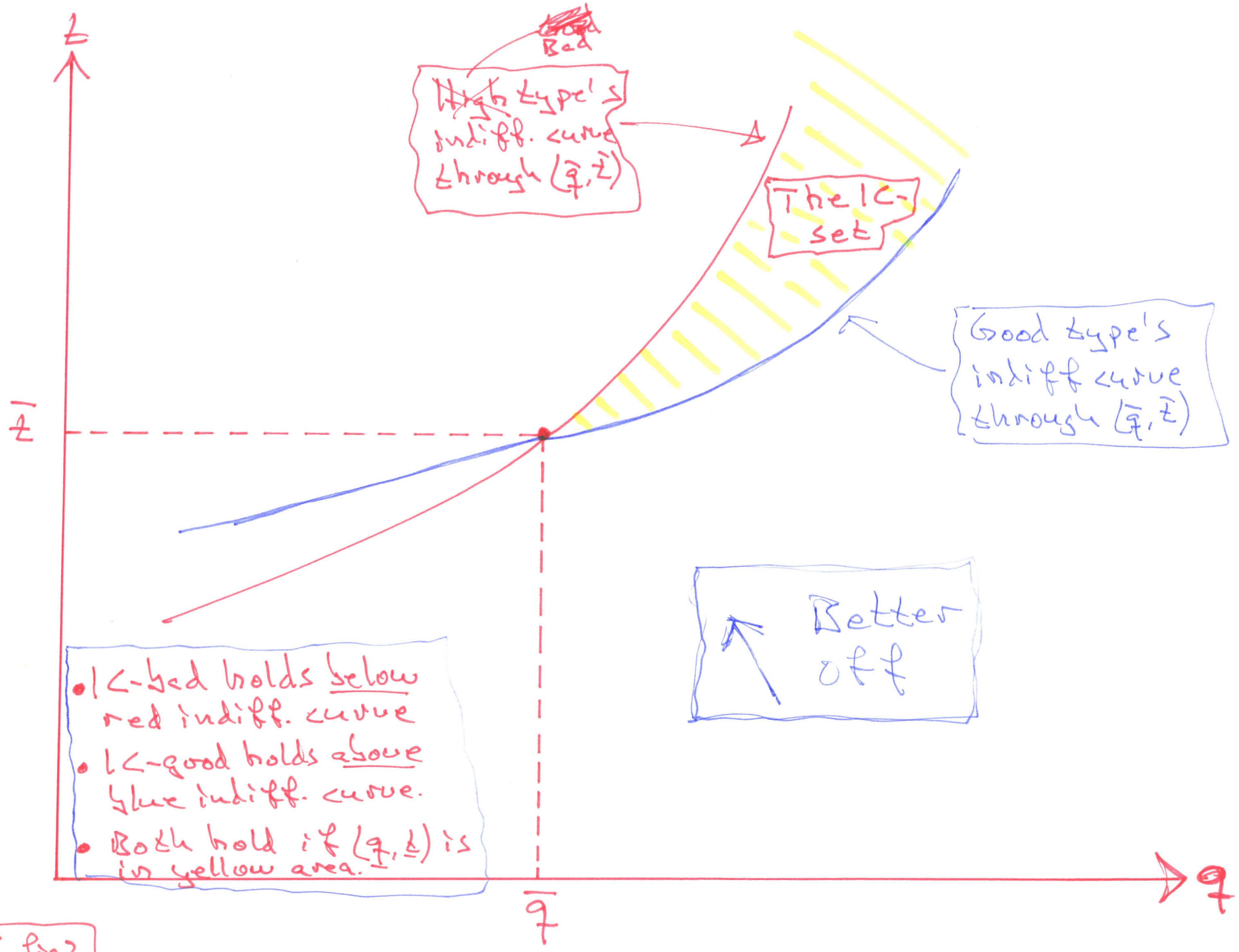


t
Bad type's indiff curve
$t = \bar{\theta} q$
The principal's indiff. curve
$t = S(q) - v$
$\bar{t}^{FB}$
B
Good type's indiff. curve,
$t = \underline{\theta} q$
$\bar{q}^{FB}$
q

LH-I, fig I

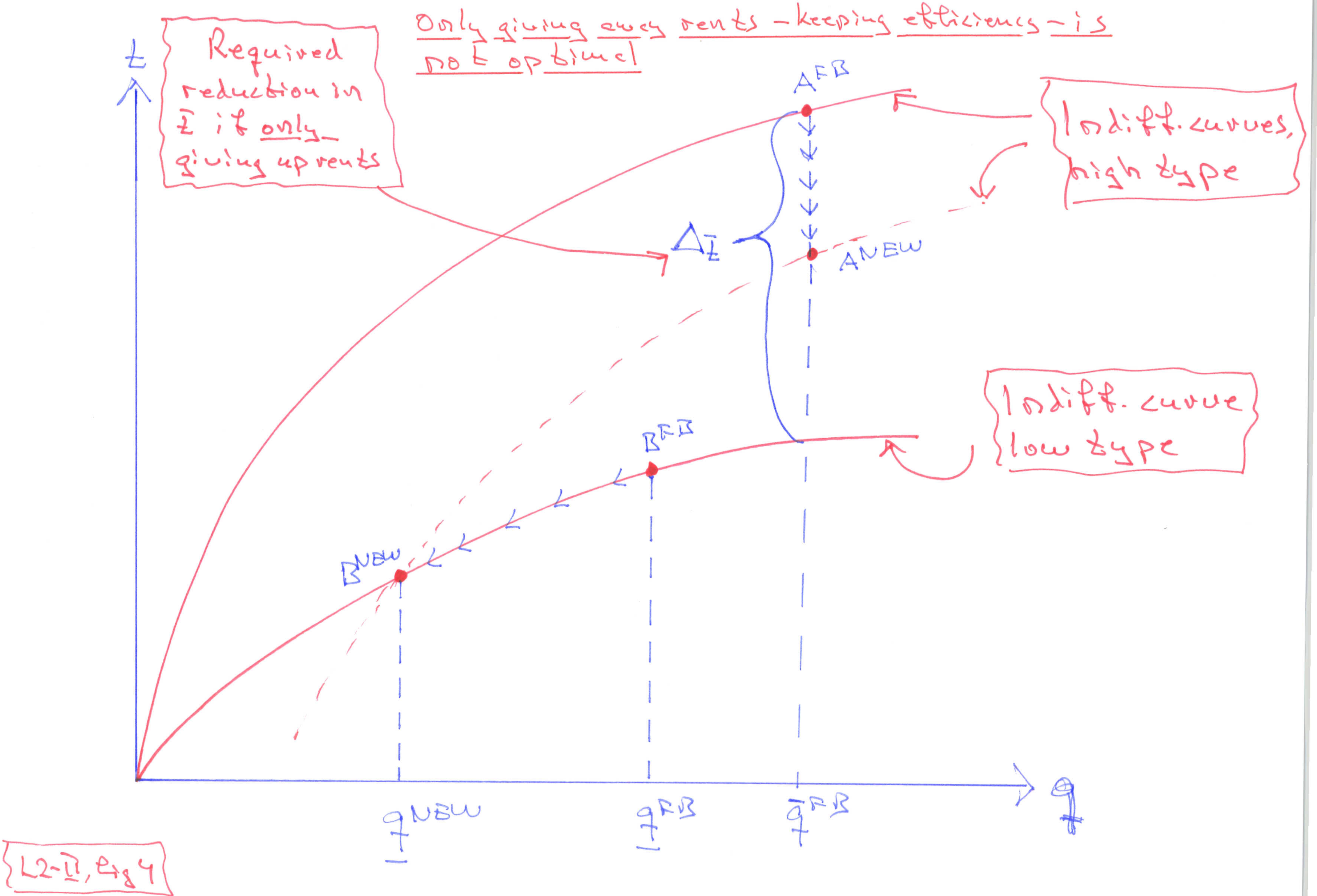

t
Slope = θ̄
Slope = s'(q)
B
A
Good type's indiff curve through B. Slope = θ
q
$\bar{q}^{FB}$

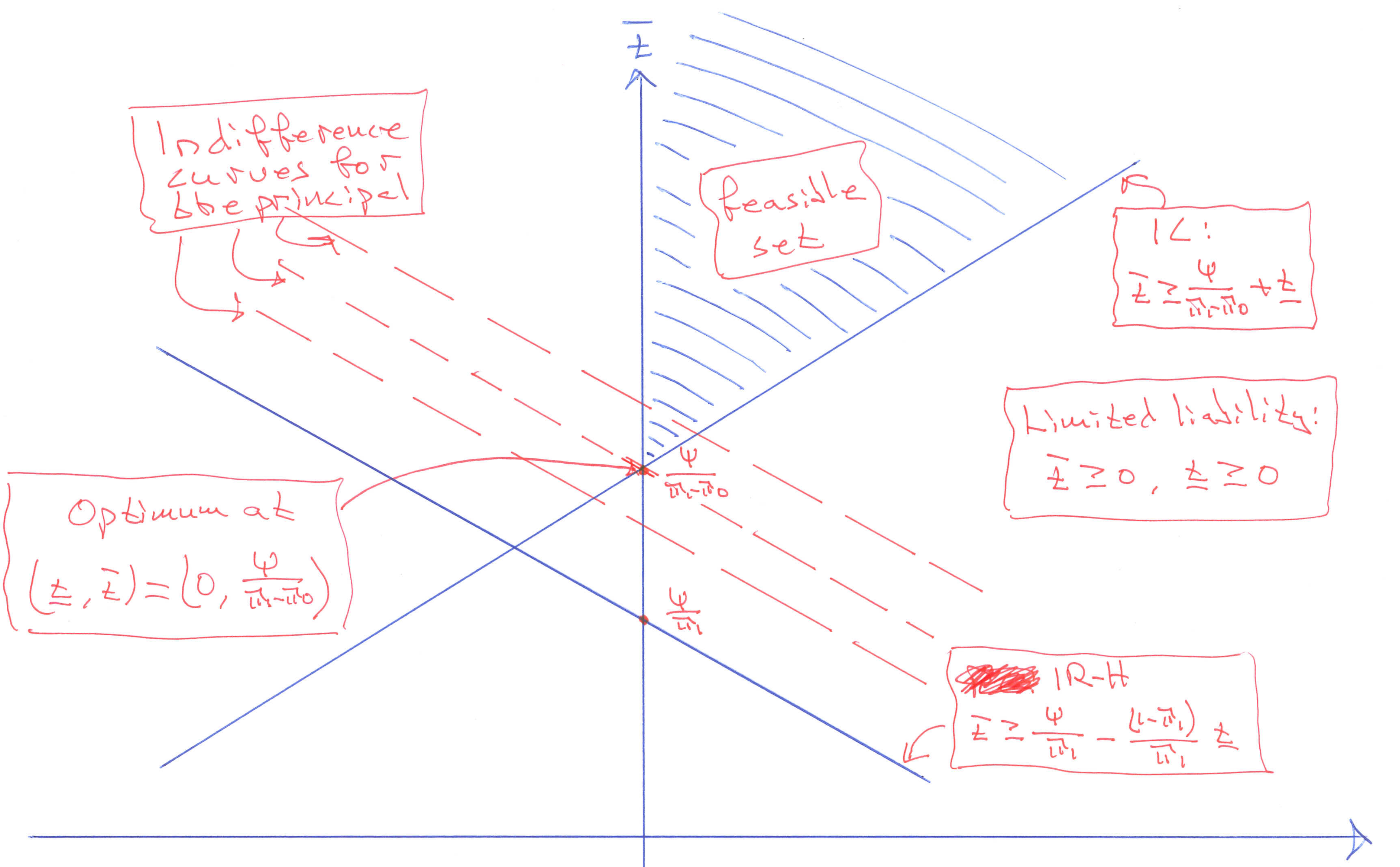
L4-I, Fig 2



t
~~High~~ Bad type's indiff. curve through $(\bar{q}, \bar{t})$
The IC-set
Good type's indiff curve through $(\bar{q}, \bar{t})$
$\bar{t}$
Better off
• IC-bad holds below red indiff. curve
• IC-good holds above blue indiff. curve.
• Both hold if (q, t) is in yellow area.
$\bar{q}$
q

L3-I, fig2


Only giving away rents – keeping efficiency – is not optimal
Required reduction in $\bar{t}$ if only giving up rents
$\Delta\bar{t}$
A^{FB}
A^{NEW}
B^{FB}
B^{NEW}
Indiff. curves, high type
Indiff. curve low type
t
q
$\underline{q}^{NEW}$
$\underline{q}^{FB}$
$\bar{q}^{FB}$
L2-II, Fig 4



L7, fig 2

Moral hazard with a risk neutral agent protected by limited liability.